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OSCILLATIONS OF SCATTERING AMPLITUDE AND RESTRICTIONS ON ITS HIGH-ENERGY BEHAVIOR

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OSCILLATIONS OF SCATTERING AMPLITUDE AND RESTRICTIONS ON ITS HIGH-ENERGY BEHAVIOR[1]

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ABSTRACT: Meyman's modified Theorem I is proved. It is used to lower the upper Froissart-Martin limit for the scattering amplitude $f(s) \equiv f(s,t \equiv 0)$ in various assumptions concerning the behavior of $H(s) \equiv \mathrm{Im} f(s)/\mathrm{Re} f(s)$ for physical values of $s \to \infty$, provided that in a strictly formulated sense there are no abnormally strong oscillations of $f(s)$. Here we make use of the analytic functionality of the amplitude only in the upper s-half-plane. In the proof we take into account that the amplitude $f(s)$ for the real values of s is a generalized rather than an ordinary function. This result supports those recently obtained by Khuri, Kinoshita and Vernov. It is extended to the case of arbitrary binary reactions. The advantages of introducing Meyman asymptotic amplitudes are pointed out.

INTRODUCTION

A few years ago, using a rather general assumption of the correctness of the Mandelstam concept, which has yet to be strictly proven, Froissart [1] obtained the upper amplitude limit[2] /30*

$$|f(s)| \leqslant O(s \ln^2 s). \tag{1}$$

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* Numbers in the margin indicate pagination in the foreign text.

[1] This study was reported at the Conference on the Axiomatic Approach to the Theory of Unit Particles, On April 4, 1968, in Kiev.

[2] In this article we will not differentiate between s = the square of the energy in the center of mass system and E = the energy in the laboratory system, as long as they coincide asymptotically at large values of s, which are the only ones that will concern us here.

Not long ago, Martin [2] succeeded in a sufficiently strict justification of this limit on the basis of the fundamental postulates of the quantum field theory, and therefore it is possible to call it absolute. Khuri and Kinoshita [3], using certain supplementary assumptions, directly verified through experiments, succeeded in obtaining various relative upper limits, by applying, besides analytic functionality and unitarity, cross-symmetry and real similarity. In particular, they showed that, if the amplitude of elastic scattering of a truly neutral scalar particle on another scalar particle satisfies the limit (1), and if, furthermore, beginning with any s_0 where $s \to \infty$ along the real axis,

$$|H(s)| \leqslant \operatorname{ctg}(\pi\alpha), \qquad 0 < \alpha \leqslant 1/2,$$
$$H(s) = \operatorname{Im} f(s)/\operatorname{Re} f(s), \tag{2}$$

(the case where $\alpha \to \frac{1}{2}$ corresponds to a predominance of Re $f(s)$ over Im $f(s)$), then, at physical values of $s \to \infty$.

$$|f(s)| < O(s^{1-\alpha/2} \ln^2 s). \tag{3}$$

Here we used the "local" method of introducing the auxiliary function and we assumed, in accordance with the requirements of Meyman's Theorem 1, the absence, as we shall say, of strong /31 oscillations of the amplitude $f(s)$ at physical values of $s \to \infty$. The exact meaning of this term is given in [3] and will be further explained in Section 1.

Very recently, Vernov [4], using the "integral" method of introducing supplementary functions and the same Meyman's Theorem 1, succeeded in yet further reinforcing this result of Khuri and Kinoshita. He showed, using the same assumptions (1) and (2) (the supplementary assumption $H(s) < 0$ contained in (4) is actually superfluous), but with no restriction on possible amplitude oscillations over an infinite sequence of physical points s_i, tending toward infinity, that

$$|f(s_i)| < O(s_i^{1-2\alpha+\varepsilon}). \tag{4}$$

However, if we use sufficiently large physical values of s, instead of (2) and the amplitude satisfies the condition,

$$\operatorname{ctg}(\pi\beta) \leqslant H(s) \leqslant \operatorname{ctg}(\pi\alpha), \qquad 0 < \beta \leqslant 1/2, \tag{5}$$

then result (4) is amplified even more:

$$|f(s_i)| < O(s_i^{-1+2\beta+\varepsilon}). \tag{6}$$

Furthermore, Khuri and Kinoshita [3], in the same assumption of the absence of strong amplitude oscillations showed that if α in (2) disappears, but at sufficiently large physical values of s,

$$|H(s)| \leqslant O(\ln^{1-\delta} s), \tag{7}$$

where $\delta > 0$ may be arbitrarily small, then

$$|f(s)| \leqslant O(s \ln^{-M} s), \tag{8}$$

where $M > 0$ is an arbitrarily high number. But in [5] it was shown that the same result for (8) is obtained even if we use far more liberal restrictions on the possible amplitude oscillations.

As long as we have serious grounds to consider [6,7] that the general principles of the quantum field theory (and even of Mandelstam's assumption) do not place any restrictions on the possible amplitude oscillations, it is essential to determine whether or not it is also impossible to obtain the upper limits (4) and (6) for all sufficiently large physical values of s when using significantly more liberal assumptions concerning possible amplitude oscillations with reference to the absence of strong oscillations. It will be shown below that this goal has been reached, with proof that in (4), for all sufficiently large physical values of s, the condition $H(s) < 0$ is unnecessary. Furthermore, we shall demonstrate that all our initial assumptions can be completely grounded in physical fact.

1. The Necessary Theorem

To begin with, we shall consider that amplitude is continuous at the border of the holomorphic region. This unjustified restriction will be wholly discarded in Section 3. In order to broaden the general initial assumption [3] of the absence of strong amplitude oscillations at physical values of $s \to \infty$, we first of all shall somewhat modify Meyman's Theorem 1.

Modified Meyman's Theorem 1. /32

(a) Let the function $q(s)$ be holomorphic in the upper s-half-plane, possibly, with a certain finite part cut off; let it be continuous in this region and at its boundary (i.e., with the part of the real axis at a sufficient distance); let it be bounded within this region by an arbitrary linear exponent[3] and let $g(s) \to 0$ when $s \to \pm\infty$ on the real axis;

(b) Let the function $g(s)$ satisfy cross-symmetry in the form

$$g^*(-s^*) = g(s), \tag{9}$$

(c) Beginning at a certain sufficiently high value of s_0.

$$|\eta(s)| \equiv |\mathrm{Im}\, g(s)/\mathrm{Re}\, g(s)| \geqslant \mathrm{tg}(\pi\alpha), \quad 0 < \alpha \leqslant 1/2, \tag{10}$$

(d) Let the transform of the upper semicircle with radius s, which is defined by the function $g'(s) = \sqrt[2\alpha]{g(s)}$, intersect the the real axis of the g'-plane at the point $u > 0$, and

(e) let there exist as many small positive values for $\varepsilon = \varepsilon(s_0)$ and $\varepsilon' = \varepsilon'(s_0)$, as for all physical values of $s > s_0$

$$\varrho(u') \geqslant \varepsilon \sqrt{u'^2 + \varepsilon' |g'(s)|^2}, \quad u \leqslant u' \leqslant u_0, \tag{11}$$

where $\varrho(u')$ = the shortest distance from the point u' to the transform of the $[s_0, s]$ segment of the real axis, which is defined by the function $g'(s)$ (cf. the Figure, where it is shown that $g'(s)$ satisfies restriction (10) for $\alpha = \frac{1}{2}$); inequality (11) can obviously be used so long as the $g(s)$ function does not oscillate, as we shall say, to an abnormally large degree.

Then, at physical values of $s \to \infty$.

$$|g(s)| \leqslant O(s^{-\alpha\varepsilon}). \tag{12}$$

This theorem is proved to be the basis of inequality (11) in Hersch's [9] inequality

$$\int_u^{u_0} \frac{du'}{\varrho(u')} \geqslant \frac{1}{2} \ln\left(\frac{s}{s_0}\right). \tag{13}$$

We point out that the exponent s in (12) depends only on ε and not on ε'.

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[3] This condition for bounding $g(s)$ with an arbitrary linear exponent can be substituted by the weaker condition (*), presented in reference [8].

Inequality (12), naturally, is less restrictive than the proof of Meyman's Theorem 1, which is derived from (12) using $\varepsilon = 1$. However Mayman's Theorem 1 requires that $g(s)$ fulfill condition (11) at $\varepsilon = 1$ (we shall say in this case that $g(s)$ is free of strong oscillations). Inequality (12) requires only that (11) be satisfied with an arbitrarily small value of $\varepsilon > 0$ [in this case we say that $g(s)$ is free of abnormally strong oscillations (cf. Figure)].

We extend this terminology also to $f(s)$ functions which do not tend to infinity at physical values of $s \to \infty$. We shall say that $f(s)$ is free of strong oscillations if it can be represented as the product of a continuous function and a $g(s)$ function satisfying (11) with $\varepsilon' = 1$. If in this definition we do not require that $\varepsilon' = 1$, then we shall speak of the absence of abnormally strong oscillations of $f(s)$. The essence of the matter is that, in order to prove (4) or respectively (6), as will be shown below, inequality (12) is sufficient, and there is no need to appeal to a stricter proof of Meyman's Theorem 1.

2. Restrictions on Amplitude /33

Now we shall indicate the general requirements which the amplitude $f(s)$ must satisfy.

1. The amplitude $f(s)$ must be holomorphic in the upper half-plane (possibly, with a certain finite part cut off) and bounded there by an arbitrary linear exponent. This condition follows both from Meyman's [10] principle of localizability and from Lomsadze and Krivskiy's [11, 8] formulation of the principle of microcausality.

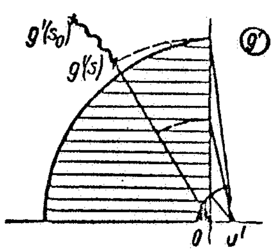

Figure: The Horizontal Lines Show the Area that is Forbidden for the $g'(s)$ Curve from Point s_0 to Point s for the Case Where $\varepsilon = \varepsilon' = 1$ (Considered in [3]); the Vertical Lines Denote the Forbidden Area in the Case Where $\varepsilon = \varepsilon' \mp \frac{1}{4}$.

2. The amplitude of $f(s)$ must be continuous in the holomorphic area and at its boundary (i.e., with the real axis at a sufficient distance). This condition together with condition 1 assures the applicability to various auxiliary functions (which will be constructed below on the basis of the $f(s)$ amplitude) of the Generalized Maximum Principle of Fragmen, Lindelef and Nevanlinna, which guarantees satisfaction of Hersch's inequality (13) and the conditions of Meyman's Theorem 1.

3. The amplitude $f(s)$ must satisfy cross-symmetry in the form

$$f^*(-s^*) = f(s), \tag{14}$$

where the left-hand side represents the analytic continuation from the upper part of the right-handed semi-axis to the upper part of the left-handed semi-axis.

In Section 3 we shall turn to the general conditions of the analytic function imposed on amplitude in order to weaken it significantly.

Then, on the basis of the Modified Meyman's Theorem 1, we can prove the following:

Theorem 1. If the amplitude $f(s)$ does not oscillate to an abnormally large degree and if it satisfies the general properties 1-3 and if inequalities (1) and (2) are correct, then at sufficiently large physical values of s

$$|f(s)| < O(s^{1-2\alpha+\delta}), \tag{15}$$

no matter how small the value of $\delta > 0$.

Proof. For the null-order iteration, let us construct (by the "local method") the auxiliary function

$$g_0(s) = f(s)s^{-1}e^{i\pi/2}(\ln s - i\pi/2)^{-\gamma}, \qquad \gamma > 2, \qquad 0 \leqslant \arg s \leqslant \pi. \tag{16}$$

The function g_0 (s) fulfills requirement (a) owing to both conditions 1 and 2 and restriction (1) for the amplitude $f(s)$. It fulfills condition (b) owing to (14) and condition (c) owing to restriction (2). One can always succeed in satisfying requirement (d) by one's choice of the sign of g_0 (s). Furthermore, as long as at large physical values of s

$$|g_0(s)| = |f(s)|s^{-1}\ln^{-\gamma} s, \tag{17}$$

since $s^{-1}\ln^{-\gamma} s$ is continuous at large values of s, requirement (d) imposes very broad restrictions on the behavior of the amplitude /3
$f(s)$ at high energies, so broad that they tend to vanish in the absence of abnormally strong oscillations.

Thus, according to the Modified Meyman's Theorem 1, at sufficiently high physical values of s

$$|g_0(s)| < O(s^{-\varepsilon\alpha}) \tag{18}$$

and consequently,

$$|f(s)| < O(s^{1-\varepsilon\alpha} \ln^{2+\delta_0} s), \tag{19}$$

where $\delta_0 > 0$ is arbitrarily small.

For the $(n + 1)$th iterational interval we shall introduce the auxiliary function

$$g_n(s) = f(s) s^{-1+\beta_{n-1}} e^{i\pi(1-\beta_{n-1})/2} (\ln s - i\pi/2)^{-\gamma}, \tag{20}$$

where

$$\beta_n = 2\alpha [1 - (1 - \varepsilon/2)^{n+1}]. \tag{21}$$

For this function

$$|\eta(s)| = |\operatorname{Im} g_n(s)/\operatorname{Re} g_n(s)| \geqslant \operatorname{tg} [\pi(\alpha - \Theta_n)], \quad \Theta_n = \alpha [1 - (1 - \varepsilon/2)^{n+1}]. \tag{22}$$

At sufficiently large values of s

$$|g_n(s)| = |f(s)| s^{-1+\beta_{n-1}} \ln^{-\gamma} s, \tag{23}$$

where $s^{-1+\beta_{n-1}} \ln^{-\gamma} s$ is continuous at large values of s. Consequently, according to the Modified Meyman's Theorem1, at sufficiently high physical values of s

$$|g_n(s)| \leqslant O(s^{-\varepsilon(\alpha-\Theta_n)}) \tag{24}$$

and this means that, in fact, at high physical values of s,

$$|f(s)| \leqslant |O(s^{1-\beta_n} \ln^{2+\delta_n} s). \tag{25}$$

Insofar as n can be arbitrarily large, there follows immediately the necessary result (15).

Here it is necessary to put special emphasis on the fact that each of the $g_n(s)$ functions must be considered free of very strong oscillations which would be capable of violating inequality (11).

It is important, however, that on account of the smoothness of the function $s^{-1+\beta_{n-1}} \ln^{-\nu} s$ at arbitrary values of n, this requirement is automatically fulfilled, if even one of these functions, let us say g_0 (s), satisfies inequality (11).

We shall show that inequality (22), strictly speaking, can be satisfied to an accuracy on the order of only $O(\ln^{-1} s)$. This indicates that in (22) we must make the substitution $\varepsilon \to \varepsilon + O(\ln^{-1} s)$. However, it is not difficult to see that the consideration of these terms does not change the result of (25) and, consequently, the final result of (15). The following theorem can be proved to be completely analogous.

Theorem II. If the amplitude $f(s)$ does not oscillate to an abnormally great degree and if it satisfies the general properties 1-3, and if inequalities (1) and (5) are accurate, then at sufficiently large physical values of s

$$|f(s)| < O(s^{-1+2\beta+\delta}). \tag{26}$$

We shall make a few remarks concerning the final result of (15) and (26). First of all, this result would not change if, instead of inequality (13), we used the weaker inequality of Nevallinna (cf. (A.9) in reference [3]). In the second place, this result does not depend either on ε, or on ε'. Thirdly, all the results obtained above are true not only at $t = 0$, but also /35
at an arbitrarily determined physical value of $t < 0$, for which the upper limit (1) is all the more correct. Finally, in the fourth place, the assumption concerning the true neutrality of one of the scattering particles is insignificant, and instead of the cross-symmetry in (14), we could use cross-symmetry in the following form (cf. for instance [13]):

$$f^{\mathrm{I}*}(-s^*) = f^{\mathrm{II}}(s), \tag{27}$$

where the exponents I and II indicate respectively the reaction and the cross-reaction. In this case it would be necessary to introduce the symmetrical and antisymmetrical amplitudes

$$f_+(s) = 2^{-1/2}[f^{\mathrm{I}}(s) + f^{\mathrm{II}}(s)],\ f_-(s) = 2^{-1/2} i [f^{\mathrm{I}}(s) - f^{\mathrm{II}}(s)], \tag{28}$$

each of which will fulfill the general properties 1-3 and in particular, the cross-relationship in the form it has in (14). The upper limit in (15) or, alternatively, (26) will then be correct for each of the amplitudes f^{I} (s) and $f^{\mathrm{II}}(s)$, if each of the amplitudes f + (s) and f - (s) does not oscillate to an abnormally great degree and if the following inequalities are satisfied:

$$|H_\pm(s)| \leqslant \operatorname{ctg}(\pi\alpha),$$

$$H_\pm(s) = \operatorname{Im} f_\pm/\operatorname{Re} f_\pm = \pm(\operatorname{Im} f^{\mathrm{I}} \pm \operatorname{Im} f^{\mathrm{II}})^{\pm 1}(\operatorname{Re} f^{\mathrm{I}} \pm \operatorname{Re} f^{\mathrm{II}})^{\mp 1} \tag{29}$$

or, alternatively, the inequalities

$$\operatorname{ctg}(\pi\beta) \leqslant H_\pm(s) \leqslant \operatorname{ctg}(\pi\alpha). \tag{30}$$

The extension of these results to the case of scattering of particles with spin does not present any problems, if we use invariant amplitudes. Let us consider, for instance, the case of πN-scattering, for which there exist four (allowing for the isotopic variables) invariant amplitudes: $A \pm (s)$ and $B \pm (s)$, which satisfy the cross-relationship (compare with [14]):

$$A^\pm(s) = \mp A^{\pm *}(-s^*), \qquad B^\pm(s) = \pm B^{\pm *}(-s^*). \tag{31}$$

The results derived by us are here applied to each of the four functions: $iA^+(s)$, $A^-(s)$, $B^+(s)$ and $iB^-(s)$, which satisfy cross-symmetry in its proper form (14).

3. Consideration of the Original Assumptions

Let us note first of all, that in order to prove all the above-indicated results, generally speaking, there was no need for the analytic function of the $f(s)$ amplitude itself. It was sufficient to use the analytic function of the asymptotic amplitude $f_\infty(s)$ assumed by Meyman [15,10], for which we use all the requirements of the Generalized Maximum Principle of Fragmen, Lindelef and Nevanlinna, which guarantees the correctness of the Modified Meyman Theorem 1. However, here it is also necessary to satisfy the condition of asymptotic equivalence of precise and asymptotic amplitudes, i.e., to satisfy inequalities [13, 12][4]

$$\sigma^\perp < O(s). \tag{32}$$

The possibility of experimentally testing this requirement was considered in [12]. We mention that the fruitful results of introducing asymptotic amplitudes, especially for non-binary reactions, /3

[4] We note that the condition of asymptotic equivalence given in [10] is not conclusive.

have been repeatedly emphasized by us (cf. [5,8,13]). The introduction of such amplitudes permits us to apply, without change, all the results of this article to the case of arbitrary non-binary reactions. Here it is significant that the Froissart-Martin upper limit (1) is correct, according to the optical theorem, also for any arbitrary non-binary reaction.

Furthermore, for the purpose of freeing ourselves of the unjustified assumption that the amplitude $f(s)$ is continuous in and at the boundary of the holomorphic region, we introduce the Modified Generalized Maximum Principle of Fragmen, Lindelef and Nevanlinna, by which we reformulate this assumption in a somewhat weaker form, in comparison with [16,17], but nonetheless on which is sufficiently applicable to high-energy physics.

We shall indicate by T the simply connected region, continuing to infinity and restricted by two half-lines Γ' and Γ'', which form an angle with the apex at the origin. Points belonging to Γ' we shall designate as ζ', points belonging to Γ'' as ζ''. Let ζ'_a and ζ''_a = points of determined sets α' and α'' having harmonic absolute or relative zero-dimensions. Let us consider the function $f(z)$ to be holomorphic within T, continuous at all $\zeta' \in \alpha'$ and all $\zeta'' \in \alpha''$ and bounded within T in the vicinity of the finite points $\zeta' \in \alpha'$ and $\zeta'' \in \alpha''$. Let us assume that in the vicinity of $z = \infty$, the function $f(z)$ is uniformly bounded within T by an arbitrary exponent of the order of α

$$|f(z)| \leqslant O(e^{\varepsilon|z|^{\alpha}}). \tag{33}$$

Then we can accept the following:

Modified Fragmen-Lindelef-Nevanlinna Generalized Maximum Principle. If $f(\zeta') \to a$ when $\zeta' \to \infty$ along an arbitrary sequence of points, excluding the points of the null-set of α', and if $f(\zeta'') \to b$ when $\zeta'' \to \infty$, then $a = b$ and $f(z) \to a$ is uniform within T.

The application of this principle to the scattering amplitude $f(s)$ (or to the nth asymptotic amplitude $f_n(s)$ [13]) is accomplished by the following steps.

1. From Meyman's [10] "Principle of Localizability of from the more liberal requirement of Lomsadze and Krivskiy's [11,8] formulation of Bogolyubov's microcausality principle we obtain the homomorphism of the amplitude $f(s)$ [or $f_n(s)$] in the upper s-half-plane and its boundedness within the upper s-half-plane by an arbitrary linear exponent. Here we make no *a priori* assumptions concerning the stage of development of the generalized functions and in particular, we can not assume that they are moderate.

2. The principle of "Amplitude Observability" [12] guarantees the boundedness of $f(s)$ [and $f_n(s)$] in the vicinity of the finite

physical points s. This principle consists of the physical requirement of the existence of "averaged amplitude"

$$f(s, \Delta s) = (f(s),\ f_{\Delta s}(s)) \tag{34}$$

as an ordinary function for any arbitrary small segment Δs containing an arbitrary physical point s. Here $f_{\Delta s}(s)$ is the basic function; it is real and non-negative-valued (let us say, "bell-shaped"), concentrated on the segment Δs, and satisfies the normalization requirement

$$\int f_{\Delta s}(s)\, ds = 1. \tag{35}$$

Due to natural physical considerations, it is required that the "averaged amplitude" be bounded for any finite physical values of s: /3'

$$|f(s, \Delta s)| \leqslant M(s), \tag{36}$$

no matter how small Δs is.

On the basis of Fatu's [18] theorem, from the boundedness of $f(s)$ in the vicinity of the finite physical points s, there follows the continuity of $f(s)$ (and $f_n(s)$) at all finite boundary points besides, possibly, sets of points having an absolute harmonic null dimension.

3. From the polynomial boundedness of $f(s)$ [or $f_n(s)$] at $s \to \infty$ along the real axis, excluding points of the null set, there follows, by virtue of the Modified Fragmen-Lindellef-Nevanlinna Generalized Maximum Principle, the amplitude's uniform polynomial boundedness in complex infinity.

4. Furthermore, in order to prove various asymptotic equations [19,13] which generalize Pomeranchuk's theorem, it is possible to apply ordinary methods [15,19], stipulating, however, the elimination of points of the null set from the real axis. Such elimination is also essential for a strict proof of all the results in this article.

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